Quarkonia potential

M. A. Durnev

Department of Theoretical Physics

St. Petersburg State University, 198904,

St. Petersburg, Russia

E-mail: mad5245@mail.ru

Abstract

Using the quark-antiquark interactions obtained in the framework of the bootstrap method we construct a potential model, investigate the possibility of describing of heavy quarkonia and calculate the bottomonium spectrum. The potential of the interaction was obtained as a nonrelativistic limit of the relativistic quark-antiquark amplitudes $Q\bar{Q} \to Q\bar{Q}$.

PACS numbers: 14.40.Lb, 14.40.Ev, 12.39.Pn

1. Introduction

Quarkonium systems, the bound states of a heavy quark and antiquark, have played a particularly important role in considering of strong interaction dynamics. The discovery of heavy quarkonia: the families of J/ψ- and Υ-mesons promoted the quark model and non-Abelian gauge field theories, leading to the prevailing picture of particle physics.

QCD features two remarkable properties. First, asymptotic freedom implies that at very high energies and momenta, quarks and gluons interact only weakly and act as quasifree particles [1, 2]. Second, confinement presumably results from the fact that at low energies the force between quarks increases with their distance, so that quarks are always tied into hadrons and cannot be removed individually.

Confinement makes it hard to calculate quantities for the bound states within QCD as one cannot apply perturbative QCD. By analogy with the positronium, and given the large masses of the charm and bottom quarks, nonrelativistic phenomenological potential models have been applied as tools for the quarkonium spectroscopy. To accommodate the properties of QCD these models, *e.g.* [3-6], are based on a short range part motivated by perturbative QCD and a phenomenological long range part accounting for confinement.

In QCD with heavy $c-$ and $b-$quarks, the characteristic scale $\Lambda_{QCD} \sim 0.2$ GeV is small as compared to the quark masses, $m_c \sim 1.5$ GeV and $m_b \sim 5$ GeV. A systematic expansion in the powers of $1/m_Q$ is possible [7-9]. The bound state problem can be dealt with non-relativistically. Following these thoughts one treats the bound state problem by solving a Schroedinger equation using an appropriate potential.

In this work the quark interaction potential is considered. The short range part of the potential is obtained as a nonrelativistic limit of the relativistic quark amplitudes of the bootstrap quark model [10, 11]. These quark amplitudes depend not only on a squared momentum transfer t, but on the energy variable s also. Therefore the direct transition to nonrelativistic potentials is not possible: these amplitudes correspond more to quasipotentials [12]. To obtain quark potentials from the quark amplitudes one has to fix the energy $s = s_0$ and then the dependence on momentum tranfer at fixed energy is considered to be potential. The energy fixing s implies an introduction of a momentum cutoff parameter Λ_F in the Fourier transformation. As a result,

the following expression is obtained for the short range part of the potential [13]:

$$V_B(r) \;=\; -\frac{1}{m_q^2}\int\limits_0^{\Lambda_\Phi}\frac{k}{r}\,\sin kr\frac{g}{1-gB(k^2)}dk, \tag{1}$$

where g is a dimensionless coupling constant, which is also a parameter of the model, $B(k^2)$ is the Chew-Mandelstam function [14] for the gluon state:

$$\begin{aligned} B(k^2) = &\left(-\beta_1\frac{k^2}{4m^2}+\beta_2\right)\sqrt{\frac{k^2+4m^2}{k^2}}\;\ln\frac{\sqrt{\dfrac{k^2+4m^2}{k^2}}+\sqrt{\dfrac{\Lambda-4m^2}{\Lambda}}}{\sqrt{\dfrac{k^2+4m^2}{k^2}}-\sqrt{\dfrac{\Lambda-4m^2}{\Lambda}}}+ \\ &+\beta_1\frac{\sqrt{\Lambda(\Lambda-4m^2)}}{4m^2}+\left(\beta_2-\beta_1\left(\frac{k^2}{4m^2}+\frac{1}{2}\right)\right)\ln\frac{1+\sqrt{\dfrac{\Lambda-4m^2}{\Lambda}}}{1-\sqrt{\dfrac{\Lambda-4m^2}{\Lambda}}}\;, \end{aligned} \tag{2}$$

where m is the mass of a heavy quark m_b, the coefficients β_1 and β_2 for 1^- state are: $\beta_1=\dfrac{1}{3}, \beta_2=\dfrac{1}{6}$.
The qualitative behaviour of the bootstrap potential with r is shown in *fig.*1. In difference with the majority of the quark interaction potentials [5, 6, 15-20] the bootstrap potential has the finite value at $r=0$ in the consequence of energy cutoff introduced in calculations of bootstrap quark amplitudes. Besides this, the presence of the momentum cutoff Λ_F leads to the origin of small oscillations of the potential at distances ~ 1 fm.
The potential of confinement is considered as a linear potential with a slope defined by the angle α. This potential is added to the bootstrap potential at a distance r_0. α and r_0 are also the parameters of our potential model. So the quarkonia potential has the form something like that shown in *fig.*2.

2. Results

The quarkonia potential considered in the previous section is used in the time-independent Schroedinger equation to find bound states, while spin-spin and spin-orbit interactions (Breit-Fermi interaction) are treated within perturbation theory. The resulting mass formula is given by

$$M(k^{2S+1}l_j) = 2m_Q + E_{kl} + \frac{32\pi\alpha_s}{9m_Q^2}\left(\frac{1}{2}S(S+1) - \frac{3}{4}\right)|\psi_{kl}(0)|^2 + \\ +\alpha_s \frac{j(j+1) - l(l+1) - S(S+1)}{m_Q^2}\left\langle \frac{1}{r^3} \right\rangle, \quad (3)$$

where α_s is a running coupling constant.

We use as input to determine the model parameters the states $\eta_b(1S)$, $\Upsilon(1S)$, $\Upsilon(2S)$ and $C(1P)$, the center of gravity for the $1P$ triplet states defined as

$$C(1P) = \frac{1}{9}(5M(\chi_{b2}) + 3M(\chi_{b1}) + M(\chi_{b0})) \approx 9900\,\mathrm{MeV}. \quad (4)$$

Identifying the $C(1P)$ with the 1^1P_1 state of the model the resulting parameter set is

$$\Lambda_F = 3.05 \text{ GeV}, \\ g = 2.22, \\ \alpha = 0.28, \quad (5) \\ r_0 = 0.39 \text{ fm},$$

the values of Λ_b (in the B-function) and m_b are taken from the bootstrap method [21].

The resulting masses are compared with the experimental data in *table* 1, while in *fig.*3 the resulting spectrum is displayed together with the experimental one. The bottomonium S-wave state reduced radial wavefunctions calculated within the model are shown in *fig.*4 and for the P- and D-wave states are shown in *fig.*5a,b.

3. Conclusion

In this work, we have costructed a potential model for heavy quarkonium based on the bootstrap potential. The aim of this work has been to investigate the possibility of describing of heavy quark-antiquark systems with the help of the derived potential using the Schroedinger equation and to get a satisfactory description of the quarkonium spectra with minimal phenomenological input.

We have demonstrated that a satisfactory description of the quarkonium spectra is possible within this model with reasonable values of the parameters. In the future one should use it to calculate other properties such as quarkonia radii, decay widths and branching ratios.

4. Acknowledgments

The author is grateful to S. M. Gerasyuta for support and discussions. This research is supported in part by Russian Ministry of Education (grant 2.1.1.68.26).

References

1. D.J. Gross, F. Wilczek, Phys. Rev. Lett. **30**, 1343 (1973).
2. H.D. Politzer, Phys. Rev. Lett.**30**, 1346 (1973).
3. A.De Rujula, H. Georgi, S.L. Glashow, Phys. Rev. D **12**, 147 (1975).
4. E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane, T.-M. Yan, Phys. Rev. D **17**, 3090 (1978).
5. J.L. Richardson, Phys. Lett. B **82**, 272 (1979).
6. W. Buchmuller, S.H.H. Tye, Phys. Rev. D **24**, 132 (1981).
7. N. Isgur, M.B. Wise, Phys. Lett. B **232**, 113 (1989).
8. H. Georgi, Phys. Lett. B **240**, 447 (1990).
9. A.F. Falk, H. Georgi, B. Grinstein, M.B. Wise, Nucl. Phys. B **343**, 1 (1990).
10. V.V. Anisovich, S.M. Gerasyuta, Sov. J. Nucl. Phys. **44**, 174 (1986).
11. V.V. Anisovich, S.M. Gerasyuta, A.V. Sarantsev, Int. J. Mod. Phys. **6**, 625 (1991).
12. A.A. Logunov, A.N. Tavkhelidze, Nuovo Cim. A **29**, 370 (1963).
13. S.M. Gerasyuta, Yu.A. Kuperin, A.V. Sarantsev, E.A. Yarevsky, Yad. Fiz. **53**, 1397 (1991).
14. G.F. Chew, *The Analytic S-Matrix*, (Benjamin, New York, 1966).
15. R.H. Dalitz, Nucl. Phys. A **353**, 251 (1981).
16. S. Ono, Preprint CERN TH.3679, 1983.
17. S.N. Gupta, S.F. Radford, W.W. Repko, Phys. Rev. D **26**, 3305 (1982).
18. I.M. Dremin, A.V. Leonidov, TMF **51**, 178 (1982).
19. A.J.D. Hey, R. Kelly, Phys. Rep. C **96**, 71 (1983).
20. A. Martin, Preprint CERN TH-4060/84, 1984; Preprint CERN TH-6604/85, 1985.
21. S.M. Gerasyuta, A.V. Sarantsev, Yad. Fiz. **52**, 1483 (1990).
22. W.-M. Yao *et al.,* J. Phys. G **33**, 1 (2006).

23. J.L. Rosner, hep-ph/0606166.

Figure and table captions

Figure 1: The bootstrap potential as a short range part of the model potential.

Figure 2: The model potential.

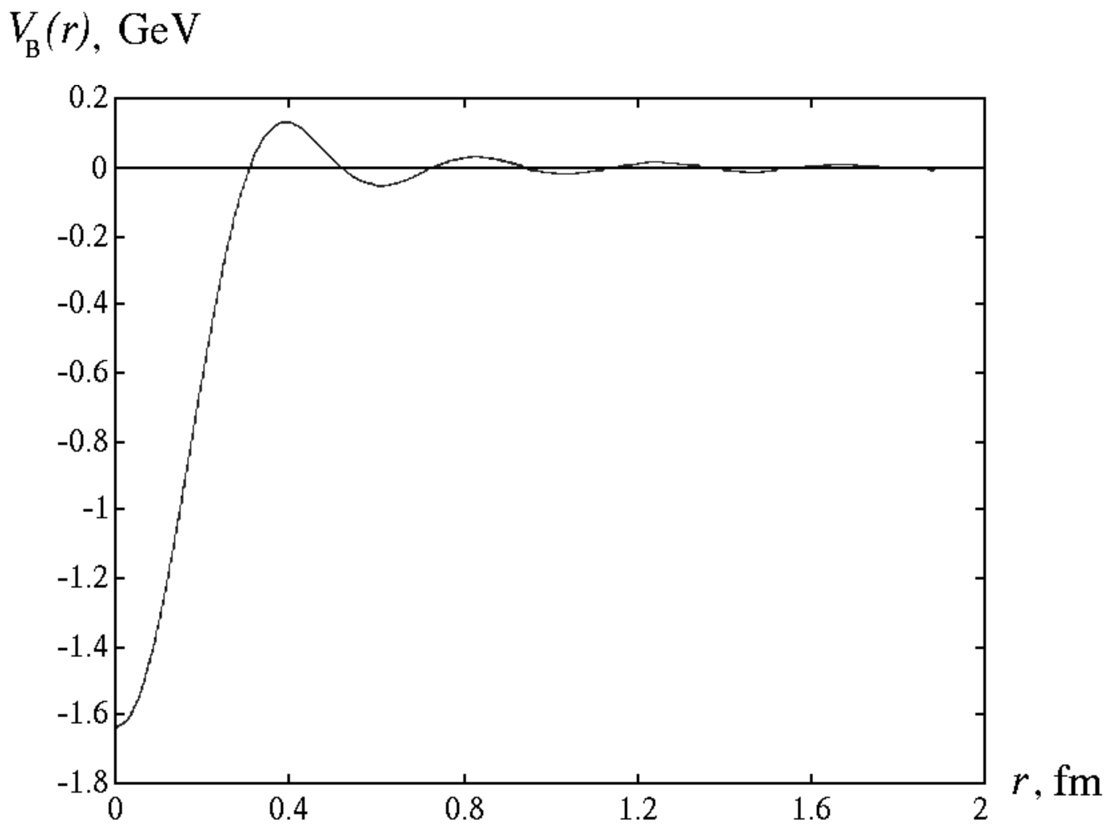
Figure 3: Bottomonium spectra of experiment and our model; $\eta_b(2S)$, $\eta_b(3S)$, $h_b(1P)$ and $h_b(2P)$ mass measurements [23] are included separately in the experimental spectrum.

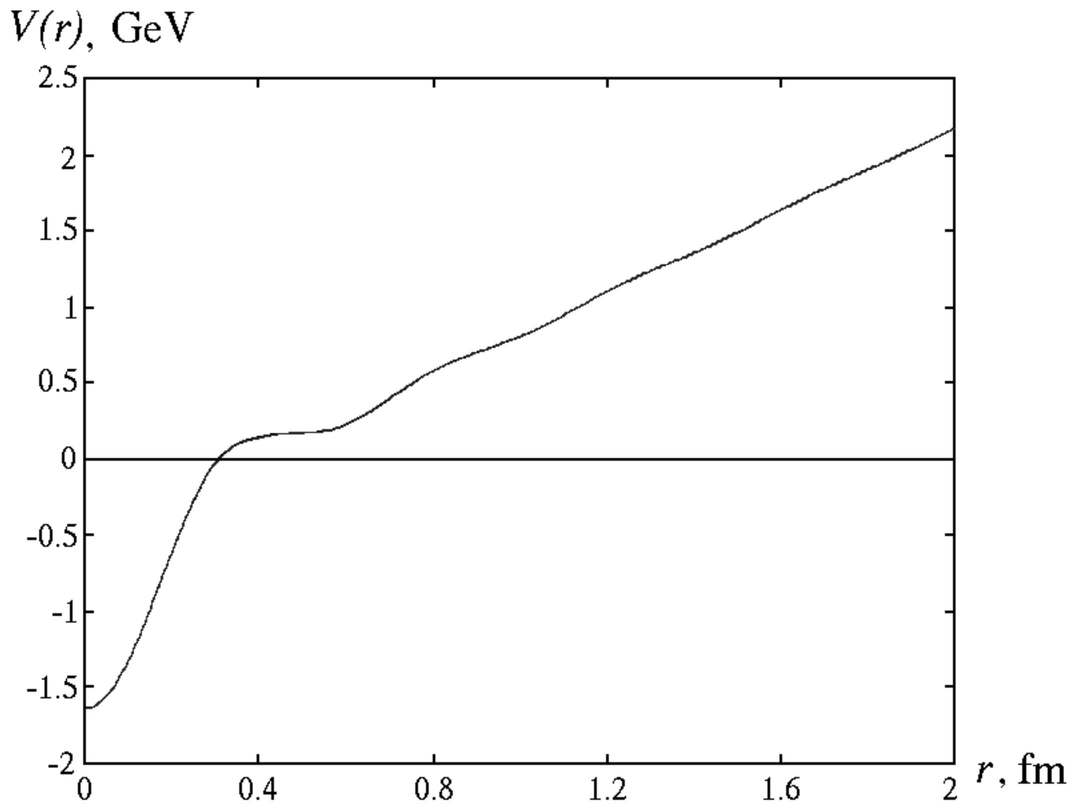
Figure 4: Bottomonium S-wave state reduced radial wavefunctions calculated within our model.

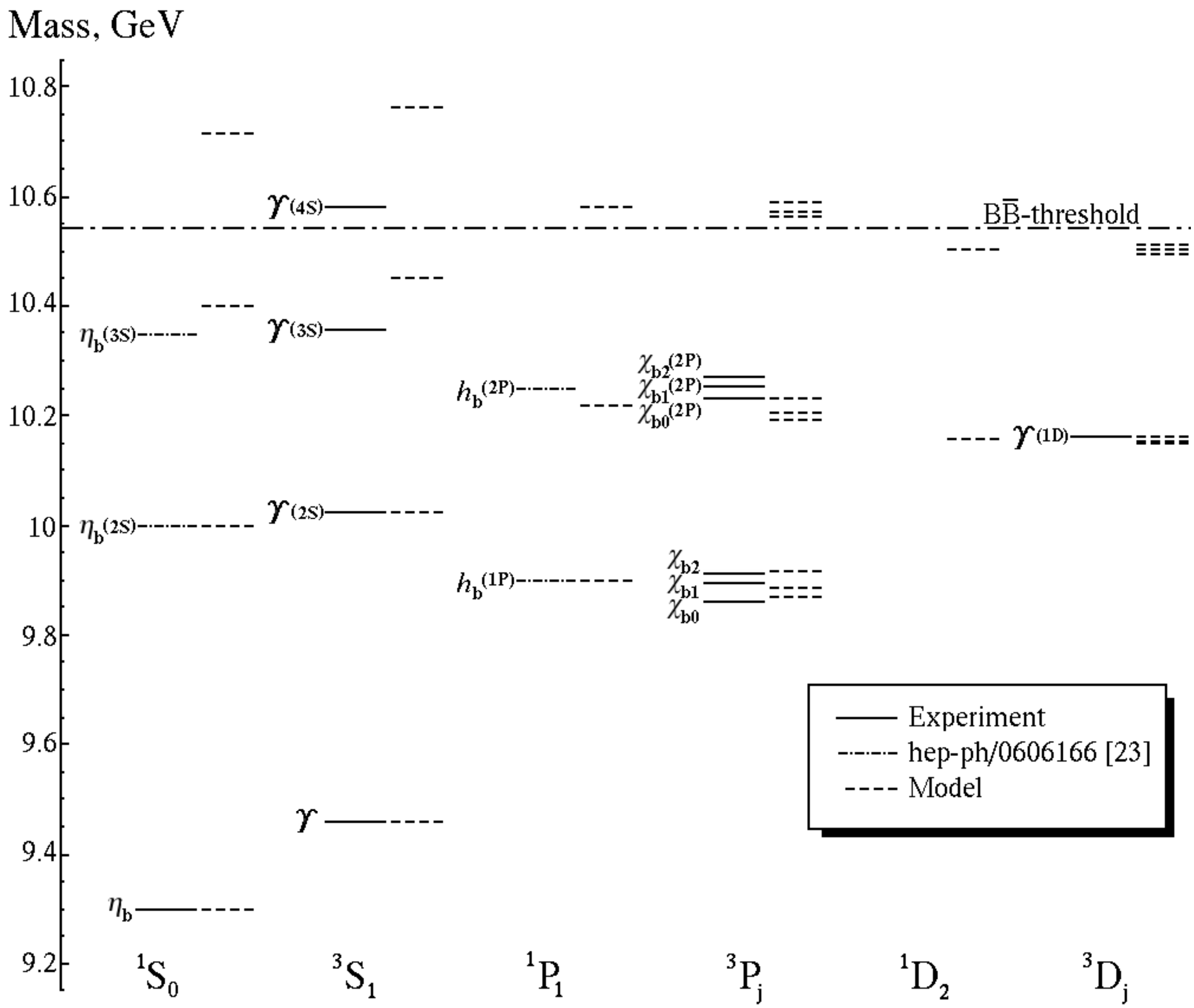
Figure 5a: Bottomonium P-wave state reduced radial wavefunctions calculated within our model.

Figure 5b: Bottomonium D-wave state reduced radial wavefunctions calculated within our model.

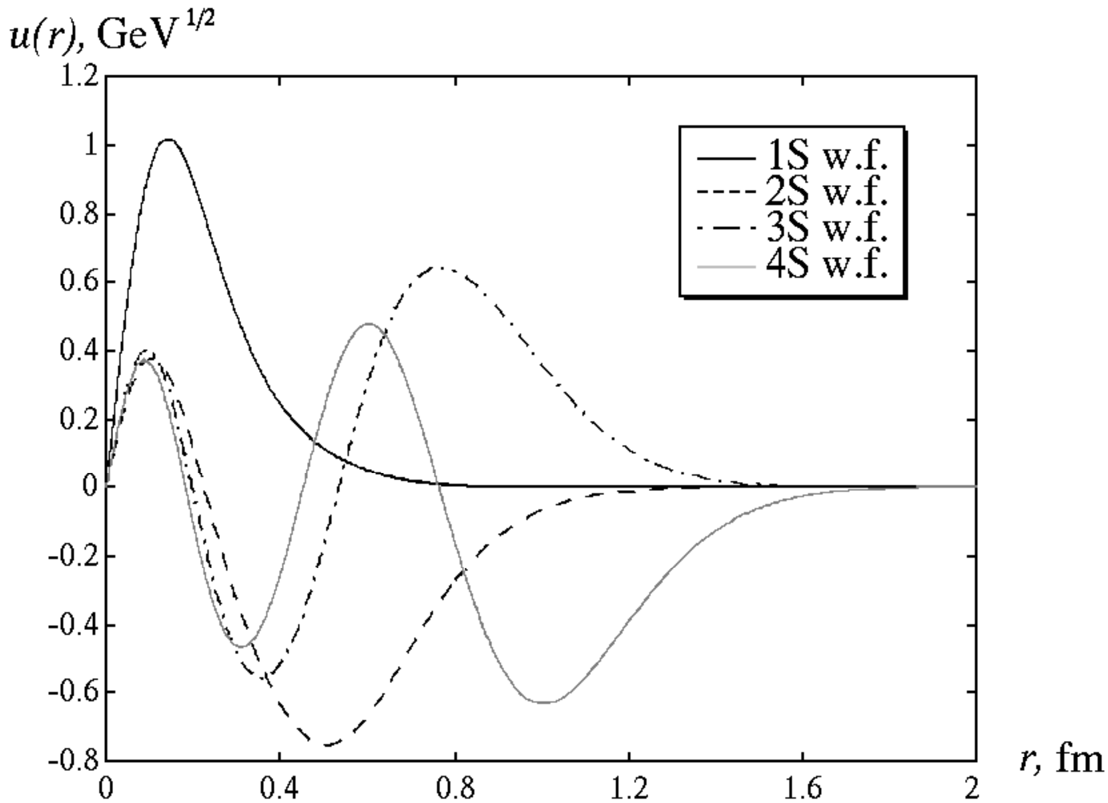
Table 1: $b\bar{b}$-state masses from the experiment and our model.

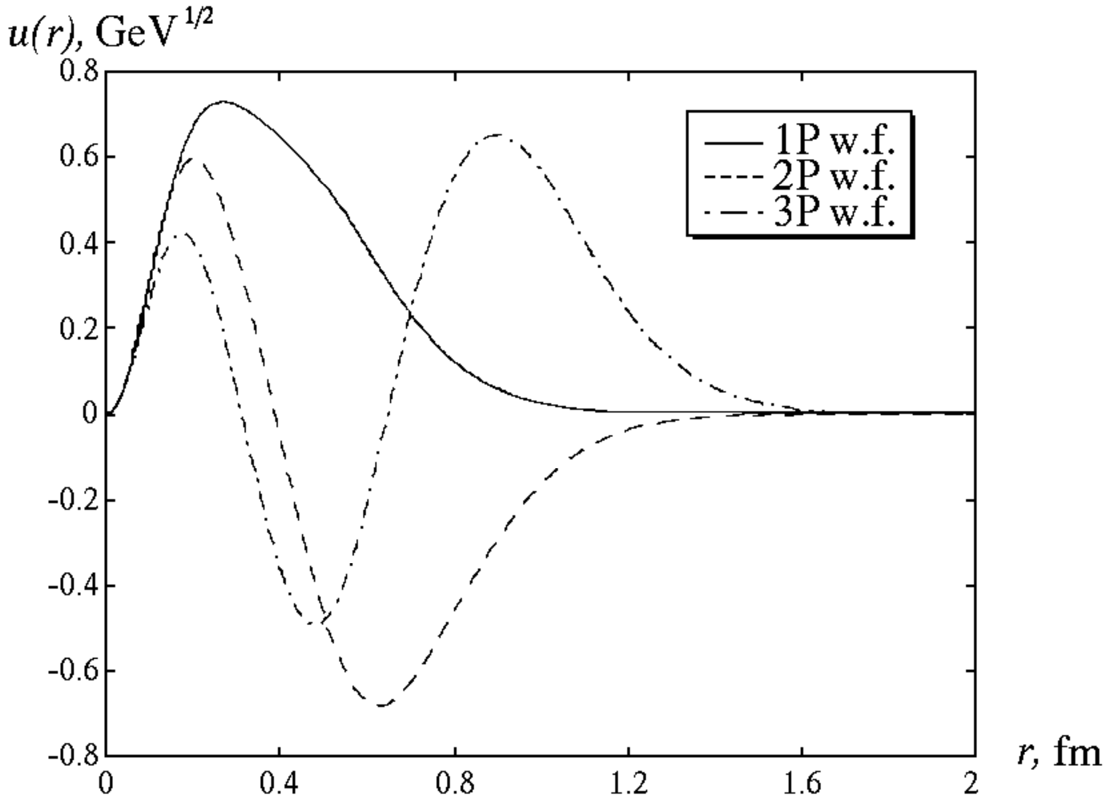

$V_B(r)$, GeV
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
0
0.4
0.8
1.2
1.6
2
r, fm


V(r), GeV
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.4
0.8
1.2
1.6
2
r, fm



Mass, GeV
10.8
10.6
10.4
10.2
10
9.8
9.6
9.4
9.2
Υ(4S)
Υ(3S)
Υ(2S)
Υ
η_b(3S)
η_b(2S)
η_b
h_b(2P)
h_b(1P)
χ_{b2}(2P)
χ_{b1}(2P)
χ_{b0}(2P)
χ_{b2}
χ_{b1}
χ_{b0}
Υ(1D)
B$\bar{\text{B}}$-threshold
Experiment
hep-ph/0606166 [23]
Model
$^1S_0$
$^3S_1$
$^1P_1$
^3P_j
$^1D_2$
^3D_j


u(r), GeV 1/2
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.4
0.8
1.2
1.6
2
r, fm
1S w.f.
2S w.f.
3S w.f.
4S w.f.


u(r), GeV 1/2
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.4
0.8
1.2
1.6
2
r, fm
1P w.f.
2P w.f.
3P w.f.

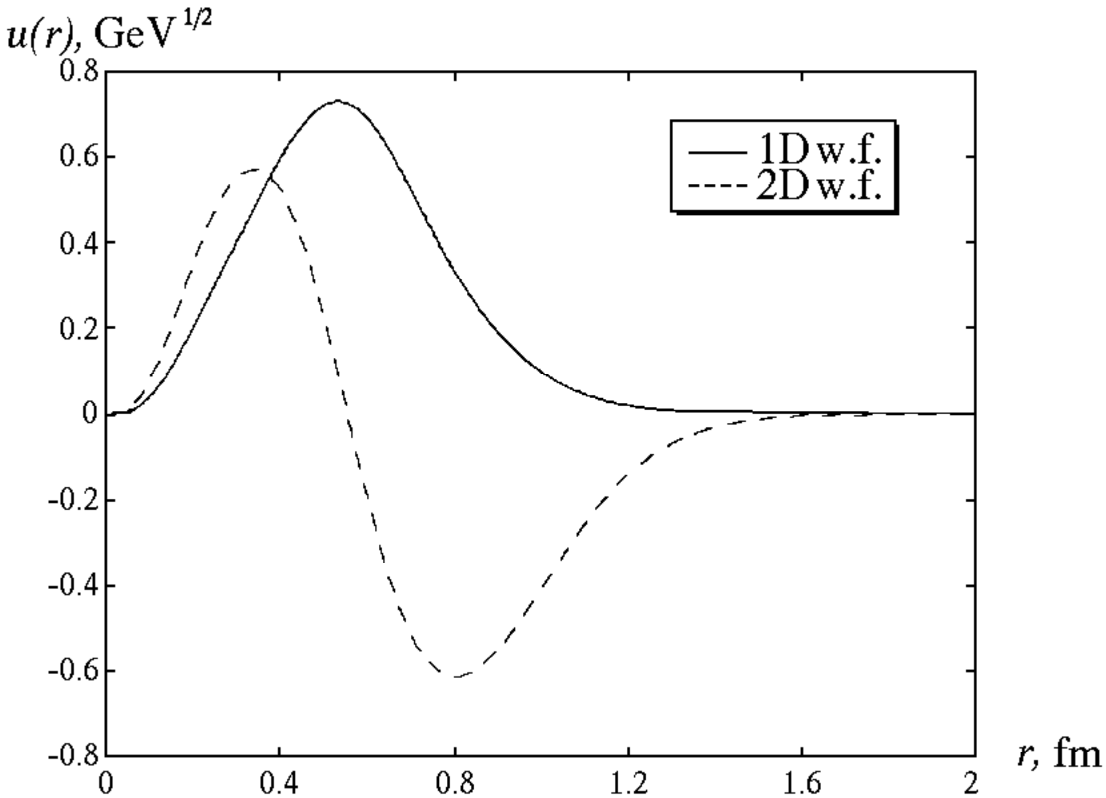

u(r), GeV 1/2
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.4
0.8
1.2
1.6
2
r, fm
1D w.f.
2D w.f.

Table 1. $b\bar{b}$-state masses from the experiment and our model.

State	Candidate	Experimental mass [22], MeV	Theoretical mass, MeV
1^1S_0	$\eta_b(1S)$	$9300 \pm 20 \pm 20$	9300
1^3S_1	$\Upsilon(1S)$	9460.30 ± 0.26	9460
1^1P_1			9900
1^3P_0	$\chi_{b0}(1P)$	$9859.44 \pm 0.42 \pm 0.31$	9869
1^3P_1	$\chi_{b1}(1P)$	$9892.78 \pm 0.26 \pm 0.31$	9884
1^3P_2	$\chi_{b2}(1P)$	$9912.21 \pm 0.26 \pm 0.31$	9916
2^1S_0			9997
2^3S_1	$\Upsilon(2S)$	10023.26 ± 0.31	10023
1^1D_2			10156
1^3D_1			10150
1^3D_2	$\Upsilon(1D)$	$10161.1 \pm 0.6 \pm 1.6$	10154
1^3D_3			10160
2^1P_1			10219
2^3P_0	$\chi_{b0}(2P)$	$10232.5 \pm 0.4 \pm 0.5$	10191
2^3P_1	$\chi_{b1}(2P)$	$10255.46 \pm 0.22 \pm 0.50$	10205
2^3P_2	$\chi_{b2}(2P)$	$10268.65 \pm 0.22 \pm 0.50$	10233
3^1S_0			10400
3^3S_1	$\Upsilon(3S)$	10355.2 ± 0.5	10450
2^1D_2			10505
2^3D_1			10495
2^3D_2			10502
2^3D_3			10511
3^1P_1			10580
3^3P_0			10562
3^3P_1			10571
3^3P_2			10589
4^1S_0			10716
4^3S_1	$\Upsilon(4S)$	10579.4 ± 1.2	10764